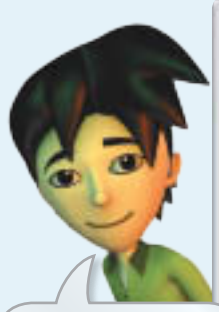
13-4 Compound Probability

Common Core State Standards

S-CP.B.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ . . . **Also S-CP.B.8, S-CP.B.9**

MP 3, MP 4, MP 6

Objective To identify independent and dependent events
To find compound probabilities



Use what you know about the locations of the two cities.

MATHEMATICAL PRACTICES

Getting Ready!

Suppose you are traveling from Philadelphia, PA, to San Diego, CA. Do you think the probability of rain in Philadelphia affects the probability of rain in San Diego? Justify your reasoning.

	Philadelphia		San Diego	
	Today	Tomorrow	Today	Tomorrow
	Rain	Sunny	Cloudy	Partly Cloudy
	43° F	58° F	70° F	75° F
	Chance of rain: 80%	Chance of rain: 0%	Chance of rain: 50%	Chance of rain: 10%

Lesson Vocabulary

- compound event
- independent events
- dependent events
- mutually exclusive events
- overlapping events

If you were to find the probability of rain in both cities in the Solve It, you would be finding the probability of a *compound event*. A **compound event** is an event that is made up of two or more events.

Essential Understanding You can find the probability of compound events by using the probability of each part of the compound event.

If the occurrence of an event does not affect how another event occurs, the events are called **independent events**. If the occurence of an event does affect how another event occurs, the events are called **dependent events**. To calculate the probability of a compound event, first determine whether the events are independent or dependent.

Think

How can you tell that two events are independent?
Two events are independent if one does not affect the other.

Problem 1 Identifying Independent and Dependent Events

Are the outcomes of each trial independent or dependent events?

A Choose a number tile from 12 tiles. Then spin a spinner.

The choice of number tile does not affect the spinner result. The events are independent.

B Pick one card from a set of 15 sequentially numbered cards. Then, without replacing the card, pick another card.

The first card chosen affects the possible outcomes of the second pick, so the events are dependent.

Got It? **1.** You roll a standard number cube. Then you flip a coin. Are the outcomes independent or dependent events? Explain.

You can find the probability that two independent events will both occur by multiplying the probabilities of each event.

take note

Key Concept **Probability of *A* and *B***

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

Problem 2 Finding the Probability of Independent Events

A desk drawer contains 5 red pens, 6 blue pens, 3 black pens, 24 silver paper clips, and 16 white paper clips. If you select a pen and a paper clip from the drawer without looking, what is the probability that you select a blue pen and a white paper clip?

Plan

Why are the events independent?
Selecting a blue pen has no affect on selecting a white paper clip.

Step 1 Let A = selecting a blue pen. Find the probability of A.

$P(A) = \frac{6}{14} = \frac{3}{7}$ 6 blue pens out of 14 pens

Step 2 Let B = selecting a white paper clip. Find the probability of B.

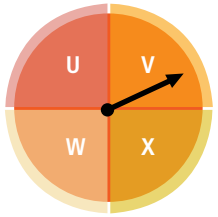
$P(B) = \frac{16}{40} = \frac{2}{5}$ 16 white paper clips out of 40 clips

Step 3 Find $P(A \text{ and } B)$. Use the formula for the probability of independent events.

$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{3}{7} \cdot \frac{2}{5} = \frac{6}{35} \approx 0.171$, or 17.1%

The probability that you select a blue pen and a white paper clip is about 17.1%.

Got It? **2.** You roll a standard number cube and spin the spinner at the right. What is the probability that you roll a number less than 3 and the spinner lands on a vowel?



Events that cannot happen at the same time are called **mutually exclusive events**. For example, you cannot roll a 2 and a 5 on a standard number cube at the same time, so the events are mutually exclusive. If events A and B are mutually exclusive, then the probability of both A and B occurring is 0. The probability that either A or B occurs is the sum of the probability of A occurring and the probability of B occurring.

take note

Key Concept **Probability of Mutually Exclusive Events**

If A and B are mutually exclusive events, then $P(A \text{ and } B) = 0$, and $P(A \text{ or } B) = P(A) + P(B)$.

Problem 3 Finding the Probability of Mutually Exclusive Events

Plan

Is there a way to simplify this problem? You can model the probabilities with a simpler problem. Suppose there are 100 athletes. In the model 28 athletes will play basketball, and 24 will be on the swim team.

Athletics **Student athletes at a local high school may participate in only one sport each season. During the fall season, 28% of student athletes play basketball and 24% are on the swim team. What is the probability that a randomly selected student athlete plays basketball or is on the swim team?**

Because athletes participate in only one sport each season, the events are mutually exclusive. Use the formula $P(A \text{ or } B) = P(A) + P(B)$.

$$P(\text{basketball or swim team}) = P(\text{basketball}) + P(\text{swim team})$$
$$= 28\% + 24\% = 52\% \qquad \text{Substitute and Simplify.}$$

The probability of an athlete either playing basketball or being on the swim team is 52%.

Got It? **3.** In the spring season, 15% of the athletes play baseball and 23% are on the track team. What is the probability of an athlete either playing baseball or being on the track team?

Overlapping events have outcomes in common. For example, for a standard number cube, the event of rolling an even number and the event of rolling a multiple of 3 overlap because a roll of 6 is a favorable outcome for both events.

Take note

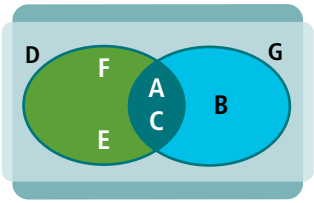
Key Concept Probability of Overlapping Events

If A and B are overlapping events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Here's Why It Works Suppose you have 7 index cards, each having one of the following letters written on it:

A B C D E F G

$P(\text{FACE})$, the probability of selecting a letter from the word FACE, is $\frac{4}{7}$.
$P(\text{CAB})$, the probability of selecting a letter from the word CAB, is $\frac{3}{7}$.



Consider $P(\text{FACE or CAB})$, the probability of choosing a letter from either the word FACE or the word CAB. These events overlap since the words have two letters in common. If you simply add $P(\text{FACE})$ and $P(\text{CAB})$, you get $\frac{4}{7} + \frac{3}{7} = \frac{4+3}{7}$. The value of the numerator should be the number of favorable outcomes, but there are only 5 distinct letters in the words FACE and CAB. The problem is that when you simply add, the letters A and C are counted twice, once in the favorable outcomes for the word FACE, and once for the favorable outcomes for the word CAB. You must subtract the number of letters that the two words have in common so they are only counted once.

$$P(\text{FACE or CAB}) = \frac{4+3-2}{7} = \frac{4}{7} + \frac{3}{7} - \frac{2}{7} = P(\text{FACE}) + P(\text{CAB}) - P(\text{AC})$$

Problem 4 Finding Probabilities of Overlapping Events

What is the probability of rolling either an even number or a multiple of 3 when rolling a standard number cube?

Know
You are rolling a standard number cube. The events are overlapping events because 6 is both even and a multiple of 3.

Need
You need the probability of rolling an even number and the probability of rolling a multiple of 3.

Plan
Find the probabilities and use the formula for probabilities of overlapping events.

Think
Why do you need to subtract the overlapping probability?
If the overlapping probability is not subtracted, it is counted twice. This would introduce an error.

$$P(\text{even or multiple of 3}) = P(\text{even}) + P(\text{multiple of 3}) - P(\text{even and multiple of 3})$$
$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$$
$$= \frac{4}{6}, \text{ or } \frac{2}{3}$$

The probability of rolling an even or a multiple of 3 is $\frac{2}{3}$.

Got It? **4.** What is the probability of rolling either an odd number or a number less than 4 when rolling a standard number cube?

Lesson Check

Do you know HOW?

1. Suppose A and B are independent events. What is $P(A \text{ and } B)$ if $P(A) = 50\%$ and $P(B) = 25\%$?

2. Suppose A and B are mutually exclusive events. What is $P(A \text{ or } B)$ if $P(A) = 0.6$ and $P(B) = 0.25$?

3. Suppose A and B are overlapping events. What is $P(A \text{ and } B)$ if $P(A) = \frac{1}{3}$; $P(B) = \frac{1}{2}$ and $P(A \text{ and } B) = \frac{1}{5}$?

Do you Understand? MATHEMATICAL PRACTICES

4. Open-Ended Give an example of independent events, and an example of dependent events. Describe how the examples differ.

5. Error Analysis Your brother says that being cloudy tomorrow and raining tomorrow are independent events. Explain your brother's error.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Determine whether the outcomes of the two actions are *independent* or *dependent* events.

See Problem 1.

6. You toss a coin and roll a number cube.

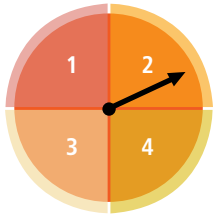
7. You draw a marble from a bag without looking. You do not replace it. You draw another marble from the bag.

8. Choose a card at random from a standard deck of cards and replace it. Then choose another card.

9. Ask a student's age and ask what year the student expects to graduate.

You spin the spinner at the right and without looking, you choose a tile from a set of tiles numbered from 1 to 10. Find each probability.

See Problem 2.



10. P(spinner lands on 2 and choose a 3)

11. P(spinner lands on an odd number and choose an even number)

12. P(spinner lands a number less than 4 and choose a 9 or 10)

A bag contains 3 blue chips, 6 black chips, 2 green chips, and 4 red chips. Use this information to find each probability if a chip is selected at random.

See Problem 3.

13. P(blue chip or black chip)

14. P(green chip or red chip)

15. P(green chip or black chip)

16. P(blue, black, or red chip)

A set of cards contains four suits (red, blue, green, and yellow). In each suit there are cards numbered from 1 to 10. Calculate the following probabilities for one card selected at random.

See Problem 4.

17. P(blue card or card numbered 10)

18. P(green or yellow card, or card numbered 1)

19. P(red card or card greater than 5)

20. P(red or blue card, or card less than 6)

B Apply

21. Pets In a litter of 8 kittens, there are 2 brown females, 1 brown male, 3 spotted females, and 2 spotted males. If a kitten is selected at random, what is the probability that the kitten will be female or brown?

22. Think About A Plan Suppose you are taking a test and there are three multiple-choice questions that you do not know the answers to. Each has four answer choices. Rather than leave the answers blank, you decide to guess. What is the probability that you answer all three questions correctly?

- Is each guess independent or dependent?
- What is the probability that a random guess answers a question when there are four answer choices?

23. Vacation In a math class, 75% of the students have visited the ocean and 50% have visited the mountains on vacation before. If 45% of the students have visited the ocean and the mountains on vacation before, what is the probability that a randomly selected student has visited the ocean or the mountains?

24. What is the probability that a standard number cube rolled three times will roll first even, then odd, and then even?

25. Writing Describe the difference between mutually exclusive and overlapping events. Give examples of each.

26. When you draw a marble out of a bag and then draw another without replacing the first, the probability of the second event is different from the probability of the first.

a. What is the probability of drawing a red marble out of a bag containing 3 red and 7 blue marbles?

b. What is the probability of drawing a second red marble if a red marble is drawn the first time and not replaced?

c. What is the probability of drawing two red marbles in a row?

Challenge

Reasoning **For each set of probabilities, determine if the events A and B are mutually exclusive. Explain.**

27. $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \text{ or } B) = \frac{2}{3}$

28. $P(A) = \frac{1}{6}$, $P(B) = \frac{3}{8}$, $P(A \text{ and } B) = 0$

29. Reasoning Are mutually exclusive events dependent or independent? Explain.

Standardized Test Prep

SAT/ACT

30. Which of the following statements is NOT true?

(A) The side lengths of an isosceles right triangle can be all whole numbers.

(B) The side lengths of a right triangle can form a Pythagorean triple.

(C) The side lengths of an equilateral triangle can be all whole numbers.

(D) The angle measures of an equilateral triangle can be all whole numbers.

Short Response

31. An arc of a circle measures 90° and is 10 cm long. How long is the circle's diameter?

32. You roll a standard number cube and then spin the spinner shown at the right. What is the probability that you will roll a 5 and spin a 3?

Mixed Review

Calculate the following permutations and combinations.

See Lesson 13-3.

33. The number of 3 letter sequences that can be made without reusing any letter.

34. The number of ways that 8 runners can finish a race, if there are no ties.

35. The number of ways a 5-member subcommittee can be formed from a 12-member student government.

Get Ready! **To Prepare for Lesson 13-5, do Exercises 36–38.**

See Lesson 13-2.

Students were asked about the number of siblings they have. The results of the survey are shown in the frequency table at the right. Find the following probabilities if a student is chosen at random from the respondents.

Number of Siblings	Frequency
0	5
1	12
2	15
3	7

36. $P(2 \text{ siblings})$

37. $P(\text{fewer than 3 siblings})$

38. $P(\text{more than 1 sibling})$